COMMON PRE-BOARD EXAMINATION 2022-23

Subject: MATHEMATICS (041)

Time Allowed: 3 Hours | Maximum Marks: 80

General Instructions:

- This question paper contains – **five sections** A, B, C, D and E. Each section is compulsory. However, there are some internal choices in some questions.
- **Section A** has **20** MCQ's of 1 mark each.
- **Section B** has **5 very Short Answer (V S A) – type** questions of 2 marks each .
- **Section C** has **6 Short Answer (S A) – type** questions of 3 marks each.
- **Section D** has **4 Long Answer (L A) – type** questions of 5 marks each
- **Section E** has **3 source based / case based / passage based / integrated units of assessment** (4 marks each) with sub parts

SECTION – A
(Multiple Choice Questions)
Each question carries 1 mark

1. If A is a square matrix such that $|A| \neq 0$ and $A^2 - A + 2I = 0$ then A^{-1} equals

   a) $I - A$ b) $\frac{1}{2}(I - A)$ c) $\frac{1}{2}(I + A)$ d) $I + A$

2. If AB = C where B and C are matrices of order 3 X 5, then the order of matrix A is

   a) 3 X 5 b) 3 X 3 c) 5 X 5 d) 5 X 3

3. The projection of $\hat{\imath} + 3\hat{\jmath} + 7\,\hat{k}$ on the vector $\widehat{7\imath} - \hat{\jmath} + 8\,\hat{k}$

   a) $\frac{60}{\sqrt{114}}$ b) $\frac{60}{114}$ c) $\frac{66}{\sqrt{114}}$ d) $\frac{114}{60}$

4. If f(x) = $\begin{cases} kx^2 & , x < 2 \\ 3 & x \geq 2 \end{cases}$ is continuous at x = 0 , the value

   a) $\frac{2}{3}$ b) $\frac{4}{3}$ c) $\frac{3}{2}$ d) $\frac{3}{4}$

5. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^5 + x\,cosx + \tan^5 x + x)\,dx$ is equal to

   a) 1 b) -1 c) 0 d) 2π

6. The degree of the differential equation $\left(1 + \left(\frac{dy}{dx}\right)^5\right) = \left(\frac{d^2y}{dx^2}\right)^2$ is

   a) 1 b) 2 c) 3 d) 4

7. The point on the half plane $2x + 3y - 12 \geq 0$ is

a) (-7 , 8) b) (7 , -8) c) (-7 , -8) d) (7 , 8)

8. If $\vec{a} . \vec{b} = |\vec{a} \times \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{3}$

9. $\int \frac{dx}{\sin^2 x \cos^2 x}$ equals

a) tanx + cotx + c b) tanx – cotx + c c) tanx . cotx + c d) sinx + cosx + c

10. The value of 'K' such that the matrix $\begin{pmatrix} 1 & k \\ -k & 1 \end{pmatrix}$ is symmetric is

a) 0 b) 1 c) -1 d) 2

11. The maximum value of Z = 3x + 4y subject to the constraints $x + y \leq 4$, $x \geq 0$, $y \geq 0$ is

a) 0 b) 12 c) 16 d) 18

12. If A is an invertible matrix of order 2 , then the det(A^{-1}) is equal to

a) det(A) b) $\frac{1}{\det A}$ c) 1 d) 0

13. If matrix A = $\begin{vmatrix} 3 - 2x & x + 1 \\ 2 & 4 \end{vmatrix}$ is singular , then x equal to

a) 0 b) 1 c) -1 d) -2

14. An urn contains nine balls of which three are red four are blue, and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different color is

a) $\frac{1}{3}$ b) $\frac{2}{7}$ c) $\frac{1}{21}$ d) $\frac{2}{23}$

15. Differential equation representing the curve y = sinx is

a) $y_2 - y_1 = 0$ b) $y_2 - y = 0$ c) $y_2 + y_1 = 0$ d) $y_1 + y = 0$

16. If $y = \tan^{-1}\frac{x}{2} - \cot^{-1}\frac{x}{2}$, then $\frac{dy}{dx}$ is

a) $\frac{4}{4+x^2}$ b) $\frac{2}{4+x^2}$ c) $\frac{4}{1+x^2}$ d) $\frac{2}{1+x^2}$

17. The angle between the vector $\vec{r} = 4\,\hat{\imath} + 8\,\hat{\jmath} + \hat{k}$ makes with the x axis is

a) $\cos^{-1}\left(\frac{13}{9}\right)$ b) $\cos^{-1}\left(\frac{13}{3}\right)$ c) $\cos^{-1}\left(\frac{\sqrt{13}}{4}\right)$ d) $\cos^{-1}\left(\frac{4}{9}\right)$

18. If the direction cosines of a line are (k , k , k) then

a) K > 0 b) 0 < k < 1 c) k = 1 d) $k = \pm\frac{1}{\sqrt{3}}$

In the following questions a statement of assertions (A) is followed by a statement of Reason (R) .Choose the correct answer out of the following choices .

a) Both A and R are true and R is the correct explanation of A
b) Both A and R are true but R is not the correct explanation of A
c) A is true but R is false
d) A is false but R is true

19. **Assertion** $\sin^{-1}(3x - 4x^3) = 3\sin^{-1}\ , x\ \ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Reason : sin 3x = 3 sinx – 4 $\sin^3$x

20. **Assertion** : If $\propto, \beta, \gamma$ are direction angles of a line then $\cos 2\propto + \cos 2\beta + \cos 2\gamma + 1 = 0$

Reason ; $\cos^2\propto + \cos^2\beta + \cos^2\gamma = 1$

SECTION B

This section comprises of very short answer type questions (VSA)of 2 marks each

21. Find the value of $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$

OR

Check whether the relation R in the set R of real numbers defined by
R = $\{(a, b) : 1 + ab > 0\}$ is reflexive , symmetric or transitive.

22. The side of an equilateral triangle is increasing at the rate of 2cm/s .At what rate is its area increasing when the side of the triangle is 20 cm.

23. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$, $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, show that $(\vec{a} - \vec{d})$ is parallel to $(\vec{b} - \vec{c})$ provided $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$

OR

Find the equation of the line passing through the point (-1 , 3 , -2) and perpendicular to the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$

24.

If y = 500 e^{7x} + 600 e^{-7x} , show that $\frac{d^2y}{dx^2} = 49\,y$

25. If $\vec{a} = 2\,\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\vec{b} = 5\,\hat{\imath} + \hat{\jmath} - \hat{k}$ represent sides of a parallelogram, then find both diagonals and a unit vector perpendicular to both diagonals of parallelogram

SECTION C

This section comprises of very short answer type questions (SA) of 3 marks each

26.

Prove that $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$

27. In answering a question on MCQ test with 4 choices per question, a student knows the answer, guesses or copies the answer. Let $\frac{1}{2}$ be the probability that he knows the answer , $\frac{1}{4}$ be the probability that he guesses and remaining he copies it . Assuming that a student , who copies the answer will be correct with the probability $\frac{3}{4}$, what is the probability that the student knows the answer ,given that he answered it correctly ?

OR

Four defective oranges are accidentally mixed with sixteen good ones and by looing at them it is not possible to differentiate between them. Three oranges are drawn at random from the lot . Find the probability distribution of X , the number of defective oranges

28. Evaluate $\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$

OR

Evaluate $\int_0^{\frac{\pi}{2}} \frac{x}{sinx+cosx}\, dx$

29. Find the general solution of the following differential equatio

$\cos^2 x \frac{dy}{dx} + y = tanx$

OR

Solve the following differential equation $\frac{dy}{dx} = 1 + x + y + xy$

30. Maximize Z = 22x + 18y subject to the constraints $x + y \leq 20$, $3x + 2y \leq 48$
$x \geq 0 \;\; y \geq 0.$

31. Evaluate $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \ , \cos^2 x}\, dx$

SECTION D

(This section comprises of long answer – type questions (L A) of 5 marks each)

32. Find the area enclosd by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

33. Show that the function $f : R \to \{x \in R : -1 < x < 1\}$ defined by f(x) = $\frac{x}{1+|x|}$, $x \in R$ is one to one and onto function

OR

Let $f : N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$ where S is the range of f is one to one and on to function

34. Find the equation of the perpendicular drawn from the point (2 , 4 , -1) to the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$. Also write down the co ordinates of the foot of the perpendicular from P to the line

Show that the two lines are $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Find also the point of intersection of these lines .

35. Solve the system of equations

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z} = 4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z} = 1\ ,\frac{6}{x}+\frac{9}{y}-\frac{20}{z} = 2$$

SECTION E

(This section comprises of 3 case study /passage based questions 4 marks each with two subparts. First two case study questions have three sub parts (i) (ii) (iii) of marks 1,1,2 respectively. The third case study question has two subparts of 2 marks each)

36. If f(x) = 2 cos 2x - cos 4x is the shape of the bridge constructed between the towns $0\ and\ \pi$situated on the hill area

a) How many turning points are there on the bridge ?
b) If critical points are the villages where the bridge is passing thrugh , between which two villages the bridge is having elevation .
c) What is the maximum height of the bridge ?
OR
Where is the lowest part of the bridge lies ?

37. A window is in the form of a rectangle surmounted by a semicircular opening . The total perimeter of the window is 10m.

a) write the formula to express the perimeter if x is the length and y is the breadth of th window
b) Write the formula to find the area of the window
c) Find the dimensions of the window to admit maximum light through the whole opening
OR
Find the least space required to have the window

38. A doctor is to visit a patient . From the pat experience , it is known that the probabilities that he will come by train , bus , scooter or by other means of transports are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$ .The probability that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$, if he comes by train , bus , scooter respectively , but if he comes by other means of transport he will not be late .

a) When he arrives he is not late , what is the total probability ?
b) When he arrives he is late . What is the probability that he comes by train